

M-Theory in the Gaugeon Formalism

Mir Faizal

Department of Mathematics, Durham University,
Durham, DH1 3LE, United Kingdom,
faizal.mir@durham.ac.uk

April 4, 2012

Abstract

In this paper we will analyse the Aharony-Bergman-Jafferis-Maldacena (ABJM) theory in $\mathcal{N} = 1$ superspace formalism. We then study the quantum gauge transformations for this ABJM theory in gaugeon formalism. We will also analyse the extended BRST symmetry for this ABJM theory in gaugeon formalism and show that these BRST transformations for this theory are nilpotent and this in turn leads to the unitary evolution of the \mathcal{S} -matrix.

1 Introduction

The action of a single membrane give by the Bergshoeff-Sezgin-Townsend (BST) has no gauge symmetry associated with it and it is generically nonconformal [1]. However, the ABJM theory that is thought to capture the dynamics of multiple $M2$ -branes is superconformal and has a gauge symmetry associated with it [2]. In fact, it is a $U(N)_k \times U(N)_{-k}$ superconformal Chern-Simons-matter theory with levels k and $-k$. It also has an arbitrary rank. Even though it explicitly has only $\mathcal{N} = 6$ supersymmetry, it is suspected that this symmetry might be enhanced to $\mathcal{N} = 8$ supersymmetry [3]-[7]. If this is done then a full $SO(8)$ R -symmetry at Chern-Simons levels $k = 1, 2$ will exist. Thus, this theory is thought to describe the dynamics of multiple $M2$ -membranes placed at the singularity of R^8/Z_k . This theory also coincides with Bagger-Lambert-Gustavsson (BLG) theory which is constructed using the only known example of a Lie 3-algebra [8]-[12]. Both the BLG theory and the ABJM theory have been analysed in the $\mathcal{N} = 1$ superspace formalism [13]-[14]. The dimensionally reduction of the ABJM theory in $\mathcal{N} = 1$ superspace formulism has been analysed [15]. In this theory a map to a Green-Schwarz string wrapping a nontrivial circle in C^4/Z_k has also been constructed.

The Fock space defined in a particular gauge in gauge theory is quite different from those in other gauges. This is because the Fock space defined in a particular gauge is not wide enough to realize the quantum gauge freedom. However, the gaugeon formalism of gauge theories provides a wider framework in which we can consider the quantum gauge transformation by introducing a set of extra fields called gaugeon fields [16]-[20]. As the ABJM theory has gauge

symmetry associated with it, the ABJM theory can also be analysed in the gaugeon formulism. This is what will be done in this paper.

The ABJM theory has been used as interesting examples of the AdS_4/CFT_3 correspondence [21]-[25]. It will be interesting to analyse the ABJM theory in $\mathcal{N} = 1$ superspace gaugeon formulism as an interesting example of AdS_4/CFT_3 correspondence because this theory will be supersymmetric without having any holomorphic property. This property of being supersymmetric without having any holomorphic property is a peculiarity of the AdS_4/CFT_3 correspondence with respect to the usual AdS_5/CFT_4 .

2 ABJM Theory

In this section we review the ABJM theory in $\mathcal{N} = 1$ superspace formulism. The classical Lagrangian density for the ABJM theory in $\mathcal{N} = 1$ superspace formulism, with the gauge group $U(N)_k \times U(N)_{-k}$, is given by,

$$\mathcal{L}_c = \mathcal{L}_M + \mathcal{L}_{CS} - \tilde{\mathcal{L}}_{CS}, \quad (1)$$

where \mathcal{L}_{CS} and $\tilde{\mathcal{L}}_{CS}$ are the Lagrangian densities for the Chern-Simons theories and \mathcal{L}_M is the Lagrangian density for the matter fields. The Lagrangian densities for the Chern-Simons theories can now be written as,

$$\begin{aligned} \mathcal{L}_{CS} &= \frac{k}{2\pi} \int d^2\theta \text{Tr} [\Gamma^a \Omega_a]_|, \\ \tilde{\mathcal{L}}_{CS} &= \frac{k}{2\pi} \int d^2\theta \text{Tr} [\tilde{\Gamma}^a \tilde{\Omega}_a]_|, \end{aligned} \quad (2)$$

where k is an integer and

$$\Omega_a = \omega_a - \frac{1}{6} [\Gamma^b, \Gamma_{ab}] \quad (3)$$

$$\omega_a = \frac{1}{2} D^b D_a \Gamma_b - \frac{i}{2} [\Gamma^b, D_b \Gamma_a] - \frac{1}{6} [\Gamma^b, \{\Gamma_b, \Gamma_a\}], \quad (4)$$

$$\Gamma_{ab} = -\frac{i}{2} [D_{(a} \Gamma_{b)} - i\{\Gamma_a, \Gamma_b\}],$$

$$\tilde{\Omega}_a = \tilde{\omega}_a - \frac{1}{6} [\tilde{\Gamma}^b, \tilde{\Gamma}_{ab}] \quad (5)$$

$$\tilde{\omega}_a = \frac{1}{2} D^b D_a \tilde{\Gamma}_b - \frac{i}{2} [\tilde{\Gamma}^b, D_b \tilde{\Gamma}_a] - \frac{1}{6} [\tilde{\Gamma}^b, \{\tilde{\Gamma}_b, \tilde{\Gamma}_a\}], \quad (6)$$

$$\tilde{\Gamma}_{ab} = -\frac{i}{2} [D_{(a} \tilde{\Gamma}_{b)} - i\{\tilde{\Gamma}_a, \tilde{\Gamma}_b\}]. \quad (7)$$

Here the super-derivative D_a is given by

$$D_a = \partial_a + (\gamma^\mu \partial_\mu)_a^b \theta_b, \quad (8)$$

and '|' means that the quantity is evaluated at $\theta_a = 0$. In component form the Γ_a and $\tilde{\Gamma}_a$ are given by

$$\begin{aligned} \Gamma_a &= \chi_a + B\theta_a + \frac{1}{2} (\gamma^\mu)_a A_\mu + i\theta^2 \left[\lambda_a - \frac{1}{2} (\gamma^\mu \partial_\mu \chi)_a \right], \\ \tilde{\Gamma}_a &= \tilde{\chi}_a + \tilde{B}\theta_a + \frac{1}{2} (\gamma^\mu)_a \tilde{A}_\mu + i\theta^2 \left[\tilde{\lambda}_a - \frac{1}{2} (\gamma^\mu \partial_\mu \tilde{\chi})_a \right]. \end{aligned} \quad (9)$$

Thus, in component form these Lagrangian densities are given by

$$\begin{aligned}\mathcal{L}_{CS} &= \frac{k}{4\pi} \left(2 \left(\epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho + \frac{2i}{3} A_\mu A_\nu A_\rho \right) \right. \\ &\quad \left. + E^a E_a + \mathcal{D}_\mu (\chi^a (\gamma^\mu)_a^b E_b) \right), \\ \tilde{\mathcal{L}}_{CS} &= \frac{k}{4\pi} \left(2 \left(\epsilon^{\mu\nu\rho} \tilde{A}_\mu \partial_\nu \tilde{A}_\rho + \frac{2i}{3} \tilde{A}_\mu \tilde{A}_\nu \tilde{A}_\rho \right) \right. \\ &\quad \left. + \tilde{E}^a \tilde{E}_a + \tilde{\mathcal{D}}_\mu (\tilde{\chi}^a (\gamma^\mu)_a^b \tilde{E}_b) \right).\end{aligned}\quad (10)$$

The Lagrangian density for the matter fields is given by

$$\mathcal{L}_M = \frac{1}{4} \int d^2 \theta \operatorname{Tr} [\nabla^a X^{I\dagger} \nabla_a X_I + \nabla^a Y^{I\dagger} \nabla_a Y_I + \mathcal{V}]_+, \quad (11)$$

where

$$\begin{aligned}\nabla_a X^I &= D_a X^I + i\Gamma_a X^I - iX^I \tilde{\Gamma}_a, \\ \nabla_a X^{I\dagger} &= D_a X^{I\dagger} - iX^{I\dagger} \Gamma_a + i\tilde{\Gamma}_a X^{I\dagger}, \\ \nabla_a Y^{I\dagger} &= D_a Y^{I\dagger} + i\Gamma_a Y^{I\dagger} - iY^{I\dagger} \tilde{\Gamma}_a, \\ \nabla_a Y^I &= D_a Y^I - iY^I \Gamma_a + i\tilde{\Gamma}_a Y^I,\end{aligned}\quad (12)$$

and \mathcal{V} is the potential term given by

$$\mathcal{V} = \frac{16\pi}{k} \epsilon^{IJ} \epsilon_{KL} [X_I Y^K X_J Y^L + Y_I^\dagger X^{K\dagger} Y_J^\dagger X^{L\dagger}]. \quad (13)$$

In this section we reviewed the ABJM theory in $\mathcal{N} = 1$ formalism. In the next section we will analyse this theory in gaugeon formalism.

3 Gaugeon Formalism

Gaugeon formalism is used to analyse quantum gauge transformations of a theory. Thus in order to analyse the ABJM in gaugeon formalism we have to first analyses the gauge symmetries associate with it. The ABJM theory in $\mathcal{N} = 1$ superspace formalism is invariant under the following finite gauge transformations,

$$\begin{aligned}\Gamma_a &\rightarrow iu \nabla_a u^{-1}, & \tilde{\Gamma}_a &\rightarrow i\tilde{u} \nabla_a \tilde{u}^{-1}, \\ X^I &\rightarrow u X^I \tilde{u}^{-1}, & X^{I\dagger} &\rightarrow \tilde{u} X^{I\dagger} u^{-1}, \\ Y^{I\dagger} &\rightarrow u Y^{I\dagger} \tilde{u}^{-1}, & Y^I &\rightarrow \tilde{u} Y^I u^{-1},\end{aligned}\quad (14)$$

where

$$\begin{aligned}u &= [\exp(i\Lambda^A T_A)], \\ \tilde{u} &= [\exp(i\tilde{\Lambda}^A T_A)].\end{aligned}\quad (15)$$

Thus, the infinitesimal gauge transformations for these fields can be written as,

$$\begin{aligned}\delta \Gamma_a &= \nabla_a \Lambda, & \delta \tilde{\Gamma}_a &= \nabla_a \tilde{\Lambda}, \\ \delta X^I &= i(\Lambda X^I - X^I \tilde{\Lambda}), & \delta X^{I\dagger} &= i(\tilde{\Lambda} X^{I\dagger} - X^{I\dagger} \Lambda), \\ \delta Y^{I\dagger} &= i(\Lambda Y^{I\dagger} - Y^{I\dagger} \tilde{\Lambda}), & \delta Y^I &= i(\tilde{\Lambda} Y^I - Y^I \Lambda).\end{aligned}\quad (16)$$

Thus, the Lagrangian for the ABJM theory is invariant under these gauge transformations

$$\begin{aligned}\delta\mathcal{L}_{ABJM} &= \delta\mathcal{L}_{kcs}(\Gamma) - \delta\tilde{\mathcal{L}}_{-kcs}(\tilde{\Gamma}) + \delta\mathcal{L}_M \\ &= 0.\end{aligned}\quad (17)$$

As ABJM theory has gauge symmetry, we have to fix a gauge before doing any calculations. This can be done by choosing the following gauge fixing conditions,

$$D^a\Gamma_a = 0, \quad D^a\tilde{\Gamma}_a = 0. \quad (18)$$

These gauge fixing conditions can be incorporate at the quantum level by adding the following gauge fixing term to the original Lagrangian density,

$$\mathcal{L}_{gf} = \int d^2\theta \operatorname{Tr} \left[b(D^a\Gamma_a) + \frac{\alpha}{2}b^2 - i\tilde{b}(D^a\tilde{\Gamma}_a) + \frac{\alpha}{2}\tilde{b}^2 \right]_|. \quad (19)$$

The ghost terms corresponding to this gauge fixing term can be written as

$$\mathcal{L}_{gh} = \int d^2\theta \operatorname{Tr} \left[\bar{c}D^a\nabla_a c - \tilde{\bar{c}}D^a\nabla_a \tilde{c} \right]_|. \quad (20)$$

In order to achieve the invariance of this theory under quantum gauge transformations, we add the following gaugeon Lagrangian density

$$\begin{aligned}\mathcal{L}_{go} &= \int d^2\theta \operatorname{Tr} \left[D^a\bar{y}D^a y + \frac{1}{2}(\bar{y} + \alpha b)^2 - D^a\bar{k}D_a k \right. \\ &\quad \left. - D^a\bar{\tilde{y}}D^a\tilde{y} - \frac{1}{2}(\bar{\tilde{y}} + \alpha\tilde{b})^2 + D^a\bar{\tilde{k}}D_a\tilde{k} \right]_|. \quad (21)\end{aligned}$$

To analyse the quantum gauge transformations, we first consider the following transformation,

$$q\alpha = \tau\alpha. \quad (22)$$

Now the gauge fields, the ghosts, the auxiliary fields and the gaugeon fields transform under quantum gauge transformations as,

$$\begin{aligned}q\Gamma^a &= \tau\nabla_a(\alpha y), & q\tilde{\Gamma}^a &= \tau\nabla_a(\alpha\tilde{y}), \\ q\bar{y} &= \tau\alpha b, & q\tilde{\bar{y}} &= \tau\alpha\tilde{b}, \\ q c &= [\tau c, \alpha y] + \tau\alpha k, & q\tilde{c} &= [\tau\tilde{c}, \alpha\tilde{y}] + \tau\alpha\tilde{k}, \\ q\bar{c} &= [\tau\bar{c}, \alpha y], & q\tilde{\bar{c}} &= [\tau\bar{c}, \alpha\tilde{y}], \\ q\bar{k} &= -\tau\alpha c, & q\tilde{\bar{k}} &= -\tau\alpha\tilde{c}, \\ q b &= [\tau b, \alpha y] - [\tau\bar{c}, \alpha k], & q\tilde{b} &= [\tau\tilde{b}, \alpha\tilde{y}] - [\tau\tilde{c}, \alpha\tilde{k}], \\ q y &= q k = 0, & q\tilde{y} &= q\tilde{k} = 0.\end{aligned} \quad (23)$$

The matter fields transform under these quantum gauge transformations as,

$$\begin{aligned}q X^I &= i(\tau\alpha y X^I - X^I \tau\alpha\tilde{y}), & q X^{I\dagger} &= i(\tau\alpha\tilde{y} X^{I\dagger} - X^{I\dagger} \tau\alpha y), \\ q Y^I &= i(\tau\alpha\tilde{y} Y^I - Y^I \tau\alpha y), & q Y^{I\dagger} &= i(\tau\alpha y Y^{I\dagger} - Y^{I\dagger} \tau\alpha\tilde{y}).\end{aligned} \quad (24)$$

The total Lagrangian density which is formed by the sum of the original Lagrangian density, the gauge fixing term, the ghost term and the gaugeon term is invariant under these quantum transformations,

$$\begin{aligned} q\mathcal{L}_t &= q\mathcal{L}_c + q\mathcal{L}_{gh} + q\mathcal{L}_{gf} + q\mathcal{L}_{go} \\ &= 0. \end{aligned} \quad (25)$$

In this section we analysed the quantum gauge transformations for the ABJM theory in gaugeon formalism. In the next section we will analyse the BRST symmetry of this theory.

4 BRST Symmetry

The BRST symmetry for gauge theories in gaugeon formalism is well understood [19]-[20]. So, we can now analyse the BRST symmetry for ABJM theory in gaugeon formalism. The BRST transformations for the gauge fields, the ghosts, the auxiliary fields and the gaugeon fields are given by

$$\begin{aligned} s\Gamma_a &= \nabla_a c, & s\tilde{\Gamma}_a &= \nabla_a \tilde{c}, \\ sc &= -\frac{1}{2}\{c, c\}, & s\tilde{c} &= \tilde{b}, \\ s\bar{c} &= b, & s\tilde{c} &= -\frac{1}{2}\{\tilde{c}, \tilde{c}\}, \\ sy &= k, & s\tilde{y} &= \tilde{k}, \\ s\bar{k} &= -\bar{y}, & s\tilde{k} &= -\tilde{\bar{y}}, \\ sb &= sk = s\bar{y} = 0, & s\tilde{b} &= s\tilde{k} = s\tilde{\bar{y}} = 0, \end{aligned} \quad (26)$$

and the BRST transformations for the matter fields are given by

$$\begin{aligned} sX^I &= i(cX^I - X^I\tilde{c}), & sX^{I\dagger} &= i(\tilde{c}X^{I\dagger} - X^{I\dagger}c), \\ sY^I &= i(\tilde{c}Y^I - Y^Ic), & sY^{I\dagger} &= i(cY^{I\dagger} - Y^{I\dagger}\tilde{c}). \end{aligned} \quad (27)$$

These BRST transformations are nilpotent

$$s^2 = 0. \quad (28)$$

The total Lagrangian density obtained by the sum of the original classical Lagrangian density, the gauge fixing term, the ghost term and the gaugeon term is also invariant under the BRST transformations,

$$\begin{aligned} s\mathcal{L}_t &= s\mathcal{L}_c + s\mathcal{L}_{gh} + s\mathcal{L}_{gf} + s\mathcal{L}_{go} \\ &= 0. \end{aligned} \quad (29)$$

As this total Lagrangian density is also invariant under the BRST transformations, so we can obtain the Noether's charge Q corresponding to the BRST transformations and use it to project out the physical state. As the BRST transformations are nilpotent, so for any state $|\phi\rangle$ we have

$$Q^2|\phi\rangle = 0. \quad (30)$$

The physical states $|\phi_p\rangle$ can now be defined as states that are annihilated by Q

$$Q|\phi_p\rangle = 0. \quad (31)$$

This criterion divides the Fock space into three parts, $\mathcal{H}_0, \mathcal{H}_1$ and \mathcal{H}_2 . The space \mathcal{H}_1 , comprises of those states that are not annihilated by Q . The space \mathcal{H}_2 comprises of those states that are obtained by the action of Q on states belonging to \mathcal{H}_1 . Due to the nilpotency of Q , all the states in \mathcal{H}_2 are annihilated by Q . The space \mathcal{H}_0 comprises of those states that are annihilated by Q and are not obtained by the action of Q on any state belonging to \mathcal{H}_1 . Clearly the physical states $|\phi_p\rangle$ can only belong to \mathcal{H}_0 or \mathcal{H}_2 . This is because any state in \mathcal{H}_0 or \mathcal{H}_2 is annihilated by Q . However, any state in \mathcal{H}_2 will be orthogonal to all physical states including itself. Thus two physical states that differ from each other by a state in \mathcal{H}_2 will be indistinguishable. So all the relevant physical states actually lie in \mathcal{H}_0 . Now if the asymptotic physical states are given by

$$\begin{aligned} |\phi_{pa,out}\rangle &= |\phi_{pa}, t \rightarrow \infty\rangle, \\ |\phi_{pb,in}\rangle &= |\phi_{pb}, t \rightarrow -\infty\rangle, \end{aligned} \quad (32)$$

then a typical \mathcal{S} -matrix element can be written as

$$\langle \phi_{pa,out} | \phi_{pb,in} \rangle = \langle \phi_{pa} | \mathcal{S}^\dagger \mathcal{S} | \phi_{pb} \rangle. \quad (33)$$

Now as the BRST are conserved charges, so they commute with the Hamiltonian and thus the time evolution of any physical state will also be annihilated by Q ,

$$Q\mathcal{S}|\phi_{pb}\rangle = 0. \quad (34)$$

This implies that the states $\mathcal{S}|\phi_{pb}\rangle$ must be a linear combination of states in \mathcal{H}_0 and \mathcal{H}_2 . However, as the states in \mathcal{H}_2 have zero inner product with one another and also with states in \mathcal{H}_0 , so the only contributions come from states in \mathcal{H}_0 . So we can write

$$\langle \phi_{pa} | \mathcal{S}^\dagger \mathcal{S} | \phi_{pb} \rangle = \sum_i \langle \phi_{pa} | \mathcal{S}^\dagger | \phi_{0,i} \rangle \langle \phi_{0,i} | \mathcal{S} | \phi_{pb} \rangle. \quad (35)$$

Since the full \mathcal{S} -matrix is unitary this relation implies that the S -matrix restricted to physical sub-space is also unitarity.

5 Conclusion

In this paper we have analysed the BRST symmetry in ABJM theory in gaugeon formalism. This allows to consider quantum gauge transformations in the ABJM theory. The BRST transformations for this theory are nilpotent and this nilpotency of these BRST transformations leads to the unitary evolution of the \mathcal{S} -matrix. This theory has a larger BRST symmetry and corresponding conserved BRST charges because apart from all the usual fields this theory also contains the gaugeon fields. In fact, the result obtained here could also have been obtained for the free part of this theory by using a conventional BRST symmetry along with the Yokoyama's subsidiary condition [26]-[27].

It may be noted that the BRST and the anti-BRST symmetries of the ABJM theory in $\mathcal{N} = 1$ superspace formulism have already been analysed [28]. Thus,

to extend this work to include anti-BRST symmetries will be straightforward. The generalization of this present work to non-linear gauges with non-linear BRST and non-linear anti-BRST symmetries might not be that straightforward. However, if this is done then we will be able to analyse the effects of ghost condensation for the ABJM theory in gaugeon formulism in these non-linear gauges. This can possibly have interesting physical consequences. So it might be interesting to extend the present work to include the BRST and the anti-BRST symmetries in the non-linear gauge.

The ABJM action for $M2$ -branes reduces to the action for $D2$ -branes by Higgs mechanics [29]-[30]. The Higgs mechanics in the gaugeon formulism has also been studied [31]. It will be interesting to study the mechanics of arriving at $D2$ -branes from $M2$ -branes in the gaugeon formulism. It is also important to analyse the effect of shift symmetry on this model. This is because a shift of fields occurs naturally in background field method. This can be done elegantly in the Batalin-Vilkovisky formalism [32]-[35]. So it might be useful to analyse this present theory in the Batalin-Vilkovisky formalism.

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